Pedro Rosario Barbosa

pmrb@coqui.net

San Juan, Puerto Rico

February 2, 2004

# Underdetermination, Logic, Mathematics and Science

## I. The Putnam-Quine Theses

One of the most controversial aspects of Philosophy today is what has to do with the problem of underdetermination in science. Most people speak of the Duhem-Quine thesis as one of its foundations. I would like to mention the fact that there is no such thing as the Duhem-Quine Thesis. As famous as this "thesis" may be, Pierre Duhem and W. V. O. Quine really stated two very different things concerning science and how its theories affect other branches completely unrelated to science.

Donald Gillies has made an excellent exposition about the similarities and differences concerning the Duhem thesis and the Quine thesis. Pierre Duhem said that in the case of Physics, an experiment can never condemn a hypothesis but a whole theoretical group. That what is really put to the test is not merely a hypothesis, but a whole bunch of hypotheses, laws and theories that the tested hypothesis supposes (Duhem 183-188; Gillies 98-99). Therefore, there is no such thing as "crucial experiments" in Physics to determine if a hypothesis is true or not. It is perfectly possible that if an experiment "disproves" a hypothesis, really the problem is not the hypothesis, but another theory, law or hypothesis, or an entire group of them within the framework of theoretical Physics (Duhem 188-190; Gillies 101). However, this underdetermination of Physics does not extend to other branches such as Medicine or Physiology. For example, if a physiologist wants to know if a nerve or a muscle has to do with the movement of the arm, all the physiologist has to do is to affect the muscle or nerve in question and will confirm if indeed it is the case. But Physics is completely different, because all physical events are understood within a theoretical framework. For example, in the area of optics, the way we explain what we observe depends not merely in what we see, but also on our theories of what light is, on the way lens work, theories about how light behaves as wave, etc. If there is any hypothesis that seems to be refuted by observation, it could be that the problem is not the hypothesis itself, but all or some elements of the theoretical body supposed by the hypothesis (Duhem 180-183). We have to add the fact that for Pierre Duhem, revision in Physics does not extend at all to the area of logic and mathematics. In fact, he was so opposed to the idea, he rejected the General Theory of Relativity and its use of non-Euclidean Geometry because it goes against our intuition that space is Euclidean (Curd and Cover 377; Gillies 105).

Quine holds a very different point of view, in “Two Dogmas of Empiricism”. He denies the distinction between analytic and synthetic judgments made by Rudolf Carnap and other logical positivists of his time. Since there is no actual distinction between formal sciences and natural sciences, he states in that essay that all of the fields of natural science and pure mathematics are convenient fictions to give meaning to experience. For Quine, all of the theoretical, logical and mathematical are there to give meaning to what we perceive through our senses. Therefore, on the basis of recalcitrant experience we can be able, not only to revise scientific theories, but also fields such as logic and mathematics (Quine 1953, 42-43).

We can see here that Quine's underdetermination of science and other fields is far stronger than Duhem's underdetermination doctrine. It is this Quine's underdetermination doctrine what has been wrongly called the Duhem-Quine Thesis.

However, this Quinean thesis is related to another one, widely known in the field of Philosophy of Mathematics and promoted by Hilary Putnam, which states that logic and mathematics can be effectively be revised by experience. We will discuss here not one but two

Putnam-Quine theses[1]:

- First Putnam-Quine Thesis: Mathematics and logic can be revised in light of experience and changes in scientific theories.
- Second Putnam-Quine Thesis: Mathematics as such exists by the fact that it is indispensable to science. (This is the so-called "indispensability argument").

## II. First Putnam-Quine Thesis

Logic and mathematics are both a priori formal sciences, and as such they always seem not related to empirical science. However, the prejudice that somehow we obtain knowledge of mathematics through sensible experience has gained much support from philosophers all over the world, even though such a point of view, when analyzed, cannot give an adequate semantical, epistemological and ontological account of mathematics[2]. The empiricist and physicalist ways of thinking presuppose no ontological existence of abstract relations and mathematical objects. Instead they imply that these are completely products of human imagination. This leads them to the First Putnam-Quine Thesis, that somehow mathematics and logic can be revised in light of experience.

One of the clearest examples of what Putnam's point is the case of Quantum Logic (Putnam 174, 248). Quine in "Two Dogmas" also alludes to Birkhoff and von Neumann and their development of Quantum Logic as a possible instance of revision of logic using empirical basis (Quine 1953, 43). There are cases in which supposedly formal classic logic does not apply, and one of these seem to be quantum mechanics. Curd and Cover give this example that concerns the principle of the excluded middle. For example, both of these formulas are

1 Jerrold Katz uses the "Quine-Putnam thesis" phrase (Katz 49-50), however, we will split in two these arguments to examine them more thoroughly.

2 Jerrold Katz has elaborated an excellent extensive analysis of these empiricist and physicalist problems in his book Realistic Rationalism.

tautologically equivalent, one can be derived from the other:

(1) $p \wedge (q_1 \vee q_2)$

(2) $(p \wedge q_1) \vee (p \wedge q_2)$

This is true in classical logic, but not so in quantum logic. For example, let's take into account the two-slit experiment. Let $p$ be "The electron is in region $R$ of the screen", $q_1$ is the proposition "The electron went through slit 1", and $q_2$ is the proposition "The electron went through slit 2". If the electron goes through slit 1 or slit 2, we don't see any wave interference pattern formed in the screen. However, if they go through both slits simultaneously, then we can see the pattern. Therefore, the inference of (1) to (2) would be invalid in quantum logic (Curd and Cover 380).

However, one philosopher who refuted this Putnam-Quine Thesis was Quine himself in his later book Philosophy of Logic. He argues that alternative logics (such as Quantum Logic) don't really revise at all classical logic. In Quantum Logic, the connectives are not defined the same way in terms of truth values as in bivalent classic logic. Hence, connectives such as negation "¬", the conjunction "∧" or implication "→" in classical logic don't mean the same thing in Quantum Logic. This can be an alternative logic alright, but it doesn't seem to revise classical logic, because the meaning of logical propositions is very different, and therefore refer to entirely different concepts. Quantum Logic would be itself another logical system besides classical logic, not its replacement (Quine 1970, 83-84). This position is very distant from the Quine of "Two Dogmas", in which he contemplates the possibility of revision of logic and mathematics altogether.

All of this is related to the issue of whether there is or there is no a priori knowledge. Though Putman is open to the idea of revisability in logic and mathematics, there can be a priori statements that seem not to be revisable. For instance, propositions like "2 + 2 = 4" leave no room for doubt at all, but propositions that seem what he calls "quasi-empirical", may be revised such as “Peano arithmetic is $10^{(20)}$ consistent.” These “quasi-empirical” statements seem to be revisable if a contradiction is found (Putnam 124-126; Fred 132-134). Putnam seems to equate revisability with empirical experience, and in his mind empirical experience somehow implies revision of a priori knowledge (Fred 135). However, Bob Hale has pointed out, a prioricity is not completely incompatible with revisability (Hale 143).

Putnam, in implicating revisability with empiricism, is not clear what the nature of this revisability in mathematics and logic. For example, Platonists and realists in general do not equate a prioricity with infallibility of knowledge. There are mathematical truths that we know infallibly, such as the theorem that the square root of two is an irrational number. But there are also some mathematical conjectures that still remain undecidable, reason by which philosophers like Philip Kitcher has rejected mathematical realism in general (Kitcher 36-48). James R. Brown does not find fallibilism incompatible with Platonism at all, nor with a priori knowledge. Brown says that the fact that we do not know if certain mathematical conjectures are true, doesn't mean that ontologically speaking they are not true nor false. If we concede that non-discovery implies non-existence, this would be equivalent say that in 1700 no one knew that there was a planet called Neptune and therefore Neptune didn't exist at that time. Though there is a priori knowledge, that doesn't mean that we have infallible a priori knowledge of everything. Fallibility in mathematics can stem from from three main sources: (1) We could formulate false conjectures, but not know they are actually false; (2) mistakes that stem from incorrect application of accepted principles (wrong calculations), and (3) the use of wrong and naïve concepts (for example, for early Greeks numbers were intrinsically related to figures and distances) (Brown 1999, 18-23). I would add a fourth aspect of fallibility in the field of mathematics, and that is that we don't know the entire abstract situation of affairs that we have not yet discovered. Notice that these sources of fallibilism have nothing to do with empirical experience, but have to do more with the way we deal with the theoretical aspect of mathematical truths as a whole. The same goes with logic.

Some people might state that there were in fact instances that science actually revised mathematics, and the General Theory of Relativity is one example of these. According to them, Einstein "proved" that space-time is non-Euclidean, while before him space was regarded as exclusively Euclidean. For them, General Relativity "proved" that Euclidean Geometry is false. First of all, non-Euclidean geometry was first formulated within mathematics, not within science. Lobachevsky, Riemann, and Bolyai developed different geometrical models that denied the "axiom" of the parallels but remained logically consistent, hence conceiving many possible geometrical spaces. Under these new geometries, Euclidean geometry was not refuted, but became one of an infinity of conceivable spaces in mathematics (Brown 1990, 101-102). And indeed this represented a revision in mathematics, but Euclidean geometry itself was not revised. It only revised the metamathematical conception of Geometry that admitted Euclidean space as the only one which is valid and possible. Also, notice that while these non-Euclidean geometries were developed, empirical criteria played absolutely no role in them. Many scientists, and mathematicians during the XIX century philosophers rejected all non-Euclidean geometry as being non-empirical because space, for them, was Euclidean. For them, simply non-Euclidean geometry was entirely false, and had no potential in being applied to science.

Poincaré put an end to this conception of non-Euclidean Geometry. He was the first one to conceive the logical possibility that a non-Euclidean Geometry could be adopted by scientists with the purpose of simplifying a theory. Poincaré dismissed this possibility because he regarded it as being highly improbable. For him, Euclidean geometry is in itself much simpler than non-Euclidean Geometry (Poincaré 72-88). It was not until Einstein, inspired by Poincaré, who adopted non-Euclidean geometry as a solution for a problem concerning the theory of relativity.

The dilemma that Einstein confronted was this:

1. If Euclidean Geometry is adopted, then it complicates the scientific theory at hand to be able to explain the relation between gravity, light speed and the consequences of Lorentz transformations to space-time.
2. If non-Euclidean Geometry is adopted, though it is more complicated than Euclidean Geometry, it simplifies the scientific theory in great measure, and can give an effective account on how gravity works and its relation to space-time (Einstein 97-126).

As we can see here, Einstein nor any other scientist "discovered empirically" that space-time is actually non-Euclidean, nor did it mean a revision in mathematics in any way. <u>The problem here was concerning mathematical models</u>. Under one model (Euclidean Geometry), the theory wouldn't work as well as in the case of the other mathematical model. In this sense, non-Euclidean geometry represented a more complicated mathematical model, but it was used to formulate a simpler theory that could account for empirical phenomena (Brown 1990, 102; Brown 1999, 86; Rosado 268). This didn't diminish at all the validity of Euclidean Geometry, just recognized the validity of non-Euclidean spaces, and its genuine application to science. In no instance we see that the validity of these models depend on empirical experience, and none of them have been refuted by it.

To sum it up, the First Putnam-Quine Thesis is false. Empirical experience cannot refute at all any mathematical or logical truths. In fact, all of these logico-mathematical truths to be proven true, they do not need at all any authority of experience. To prove the irrationality of the square root of two, or to prove the Pythagorean theorem, we only need to use mathematical notions (which themselves are not found in experience) like numbers, points, lines, figures, etc. and axioms and theorems, such as the properties of division and multiplication, or the addition of the angles of a triangle in Euclidean space, etc. Sensible experience, nor does any scientific notion (such as mass, electrons, light, etc.) play any role in these demonstrations. Even if we want to accept deviant logics based on scientific data, such as Quantum Logic, they wouldn't imply any kind of revision to logic at all. So, mathematics is a field completely independent from science, as logic is its own field also. Mathematics and logic together constitute a priori knowledge, and they are never refuted a posteriori.

I wish to add also, for those who argue the adoption of Quantum Logic on empirical grounds, that in reality Quantum Logic has not served at all to explain the behavior of quanta, light and subatomic particles. All it does apparently is to shift the mystery from Quantum Physics to Logic, but unlike non-Euclidean Geometry and General Relativity, such a Quantum Logic as a logical model has not provided any more explanatory value to science (Curd and Cover 380). It is on the grounds of logic that many scientists and philosophers have rejected the Copenhagen interpretation of Quantum Physics and contemplate the possibility of another theory of quanta we have not yet formulated.

## III. The Second Putnam-Quine Thesis: The Indispensability Argument

The Second Putnam-Quine Thesis is that mathematics is meaningful in the fact that it is indispensable to science and that if it was not indispensable to science, mathematics would have no reason to be. This seems to follow from the First Putnam-Quine Thesis, that somehow mathematics and logic are revised in light of recalcitrant experience. Even though we have refuted this at length, I wish to point to some strange consequences of the indispensability argument.

Rosado Haddock says that if mathematics is subordinated to physics, it is strange that mathematics does not at all refer to physical entities or theories of any kind. In fact it seems that mathematics most of the time is self-evident and true in any possible world, while Physics is not.

> Now, although applicable to the physical (and other) sciences, mathematical theorems seem to be true even if all actually accepted physical theories were false and, thus, the claim that only after the advent of modern physical science can we argue that mathematical theorems are true seems really amazing, to say the least. It is also extremely unreasonable to think that before the advent of modern physical science there was no way to establish the existence of mathematical entities, thus, e.g., that there exists an immediate successor of 3 in the natural number series. Moreover, it is perfectly conceivable that there exists a world in which all mathematical theorems known to present-day mathematicians are true (supposing that current mathematics is consistent), and that mathematicians know as much mathematics as they actually know, but in which none of the physical laws accepted as true nowadays were known to humanity. What is not possible is a world in which physical science were as developed as it actually is, but in which our present mathematical theories (especially those applicable to present-day physical science) were not valid, or, at least, were not considered to be valid (Rosado 269).

Katz also makes his criticisms along this line, that we can establish the existence of these mathematical entities even without empirical science (Katz 50-51). So, can we really remain with a straight face when we state that the validity of mathematics depends on the validity of scientific theories? As we have shown before, it seems the other way around. Mathematics and logic provide the theoretical basis and models for science to be able to formulate theories about the natural world. They are in every sense <u>a priori</u>, prior to any knowledge, they are the condition of possibility for any science about the world.

## IV. Conclusions

This article centers around the true relationship between formal a priori sciences such as logic and mathematics with natural sciences such as Physics. These three disciplines are independent of each other, but despite this, they all have decisive impact on each other. Logic is a sister discipline with mathematics, both formal sciences together are the basis of all formal investigations in mathematics, such as set theory, the model theory, among others. The mathematical and logical truths discovered in these a priori sciences constitute the basis on which we build scientific theories. Physics provides the clearest example of how this occurs, how we assign as variables physical notions and we can discover new relations among those notions using mathematical properties.

I cannot stress enough the independence of these fields, but we cannot stress enough also their inter-dependency. Logic and mathematics are their own fields, have their own investigation, and their ways of proceeding are completely different from that of natural science. These a priori fields do not need the authority of experience for them to be true. And natural science proceeds in a very different way, because each theory, law and hypothesis does require the authority of sensible experience. None of these disciplines can be reduced in terms of importance to the other. They are all important, and they all provide the basis for our knowledge of the world.

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